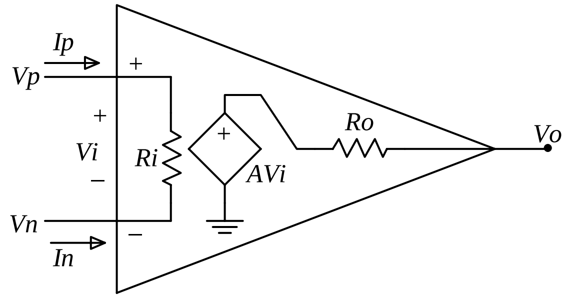
Operational Amplifier Circuits

Review:

<u>Ideal Op-amp in an open loop configuration</u>



An ideal op-amp is characterized with infinite open–loop gain

$$A \to \infty$$

The other relevant conditions for an ideal op-amp are:

1. $Ip = In = 0$
2. $Ri = \infty$
3. $Ro = 0$

<u>Ideal op-amp in a negative feedback configuration</u>

When an op-amp is arranged with a negative feedback the ideal rules are:

1. $Ip = In = 0$: input current constraint

2. $Vn = Vp$: input voltage constraint

These rules are related to the requirement/assumption for large open-loop gain $A \to \infty$, and they form the basis for op-amp circuit analysis.

The voltage *Vn* tracks the voltage *Vp* and the "control" of *Vn* is accomplished via the feedback network.

Operational Amplifier Circuits as Computational Devices

So far we have explored the use of op amps to multiply a signal by a constant. For the inverting amplifier the multiplication constant is the gain $-\frac{R2}{R1}$ and for the non inverting amplifier the multiplication constant is the gain $1+\frac{R2}{R1}$. Op amps may also perform other mathematical operations ranging from addition and subtraction to integration, differentiation and exponentiation.[1] We will next explore these fundamental "operational" circuits.

Summing Amplifier

A basic summing amplifier circuit with three input signals is shown on Figure 1.

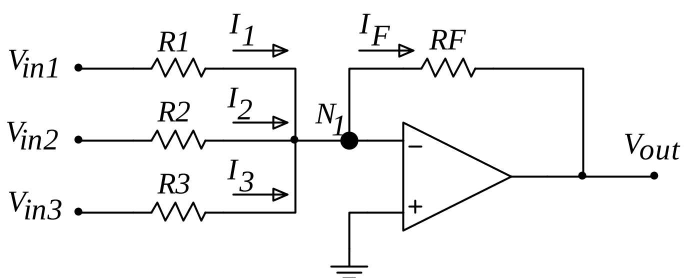


Figure 1. Summing amplifier

Current conservation at node N_1 gives

$$I_1 + I_2 + I_3 = I_F \tag{1.1}$$

By relating the currents I_1, I_2 and I_3 to their corresponding voltage and resistance by Ohm's law and noting that the voltage at node N_1 is zero (ideal op-amp rule) Equation (1.1) becomes

$$\frac{V_{in1}}{R1} + \frac{V_{in2}}{R2} + \frac{V_{in3}}{R3} = -\frac{V_{out}}{RF} \tag{1.2}$$

[1] The term operational amplifier was first used by John Ragazzini et. al in a paper published in 1947. The relevant historical quotation from the paper is:
"As an amplifier so connected can perform the mathematical operations of arithmetic and calculus on the voltages applied to its inputs, it is hereafter termed an 'operational amplifier'."
John Ragazzini, Robert Randall and Frederick Russell, " Analysis of Problems in Dynamics by Electronics Circuits," Proceedings of IRE, Vol. 35, May 1947

And so V_{out} is

$$V_{out} = -\left(\frac{RF}{R1} V_{in1} + \frac{RF}{R2} V_{in2} + \frac{RF}{R2} V_{in3} \right) \tag{1.3}$$

The output voltage V_{out} is a sum of the input voltages with weighting factors given by the values of the resistors. If the input resistors are equal $R1=R2=R3=R$, Equation (1.3) becomes

$$V_{out} = -\frac{RF}{R}\left(V_{in1} + V_{in2} + V_{in3} \right) \tag{1.4}$$

The output voltage is thus the sum of the input voltages with a multiplication constant given by $\frac{RF}{R}$. The value of the multiplication constant may be varied over a wide range and for the special case when $RF = R$ the output voltage is the sum of the inputs

$$V_{out} = -\left(V_{in1} + V_{in2} + V_{in3} \right) \tag{1.5}$$

The input resistance seen by each source connected to the summing amplifier is the corresponding series resistance connected to the source. Therefore, the sources do not interact with each other.

Difference Amplifier

This fundamental op amp circuit, shown on Figure 2, amplifies the difference between the input signals. The subtracting feature is evident from the circuit configuration which shows that one input signal is applied to the inverting terminal and the other to the non-inverting terminal.

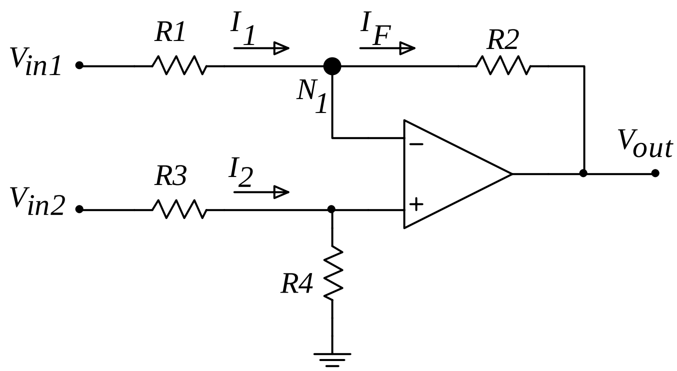


Figure 2. Difference Amplifier

Before we proceed with the analysis of the difference amplifier let's think about the overall behavior of the circuit. Our goal is to obtain the difference of the two input signals $(V_{in2} - V_{in1})$. Our system is linear and so we may apply superposition in order to find the resulting output. We are almost there once we notice that the contribution of the signal V_{in2} to the output is

$$V_{out2} = V_{in2}\left(\frac{R4}{R3+R4}\right)\left(1+\frac{R2}{R1}\right) \tag{1.6}$$

and the contribution of signal V_{in1} is

$$V_{out1} = -V_{in1}\left(\frac{R2}{R1}\right) \tag{1.7}$$

And the output voltage is

$$V_{out} = V_{out2} - V_{out1} = V_{in2}\left(\frac{R4}{R3+R4}\right)\left(1+\frac{R2}{R1}\right) - V_{in1}\frac{R2}{R1} \tag{1.8}$$

Note that in order to have a subtracting circuit which gives $V_{out}=0$ for equal inputs, the weight of each signal must be the same. Therefore

$$\left(\frac{R4}{R3+R4}\right)\left(1+\frac{R2}{R1}\right)=\frac{R2}{R1} \tag{1.9}$$

which holds only if

$$\frac{R4}{R3}=\frac{R2}{R1} \tag{1.10}$$

The output voltage is now

$$V_{out}=\frac{R2}{R1}\left(V_{in2}\text{-}V_{in1}\right) \tag{1.11}$$

which is a difference amplifier with a differential gain of *R2/R1* and with zero gain for the common mode signal. It is often practical to select resistors such as *R4=R2* and *R3=R1*.

The fundamental problem of this circuit is that the input resistance seen by the two sources is not balanced. The input resistance between the input terminals A and B, the **differential input resistance, R_{id}** (see Figure 3) is $R_{id}\equiv\frac{V_{in}}{I}$

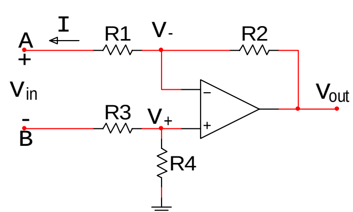


Figure 3. Differential amplifier

Since $V_+ = V_-$, $V_{in}=R1\,I\;+\;R3\,I$ and thus $R_{id}=2R1$. The desire to have large input resistance for the differential amplifier is the main drawback for this circuit. This problem is addressed by the instrumentation amplifier discussed next.

Instrumentation Amplifier

Figure 4 shows our modified differential amplifier called the instrumentation amplifier (IA). Op amps *U1* and *U2* act as voltage followers for the signals V_{in1} and V_{in2} which see the infinite input resistance of op amps *U1* and *U2*. Assuming ideal op amps, the voltage at the inverting terminals of op amps *U1* and *U2* are equal to their corresponding input voltages. The resulting current flowing through resistor *R1* is

$$I_1 = \frac{V_{in1} - V_{in2}}{R1} \tag{1.12}$$

Since no current flows into the terminals of the op amp, the current flowing through resistor *R2* is also given by Equation (1.12).

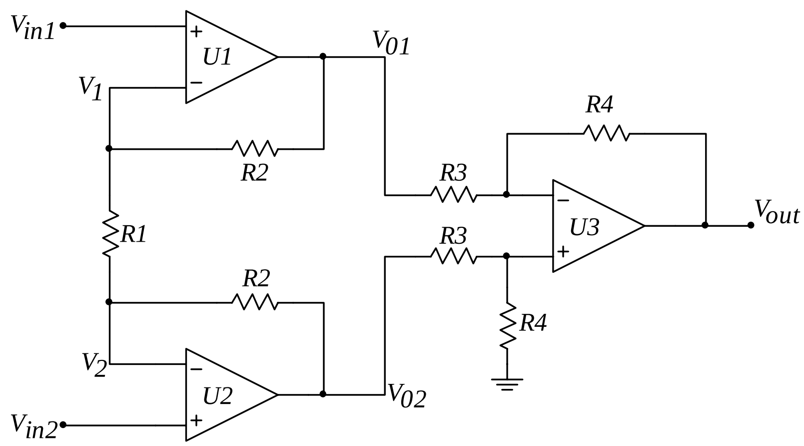


Figure 4. Instrumentation Amplifier circuit

Since our system is linear the voltage at the output of op-amp *U1* and op-amp *U2* is given by superposition as

$$\mathrm{V}_{01} = \left(1 + \frac{R2}{R1}\right)\mathrm{V}_{\mathrm{in1}} - \frac{R2}{R1}\mathrm{V}_{\mathrm{in2}} \tag{1.13}$$

$$\mathrm{V}_{02} = \left(1 + \frac{R2}{R1}\right)\mathrm{V}_{\mathrm{in2}} - \frac{R2}{R1}\mathrm{V}_{\mathrm{in1}} \tag{1.14}$$

Next we see that op amp *U3* is arranged in the difference amplifier configuration examined in the previous section (see Equation (1.11)). The output of the difference amplifier is

$$\mathrm{V}_{\mathrm{out}} = \frac{R4}{R3}\left(1 + \frac{2R2}{R1}\right)\left(\mathrm{V}_{\mathrm{in2}} - \mathrm{V}_{\mathrm{in1}}\right) \tag{1.15}$$

The differential gain, $\frac{R4}{R3}\left(1 + \frac{2R2}{R1}\right)$, may be varied by changing only one resistor: *R1*.

Current to voltage converters

A variety of transducers produce electrical current in response to an environmental condition. Photodiodes and photomultipliers are such transducers which respond to electromagnetic radiation at various frequencies ranging from the infrared to visible to γ-rays.

A current to voltage converter is an op amp circuit which accepts an input current and gives an output voltage that is proportional to the input current. The basic current to voltage converter is shown on Figure 5. This circuit arrangement is also called the *transresistance amplifier.*

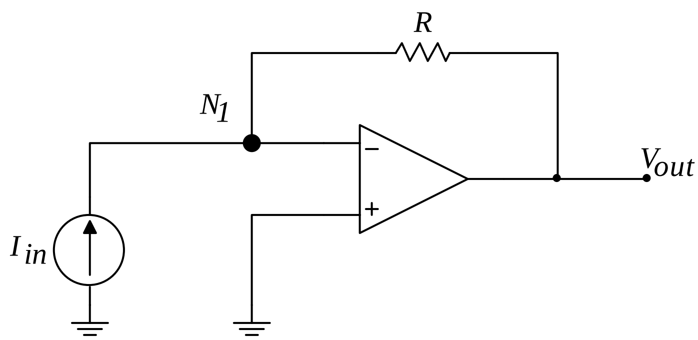


Figure 5. Current to voltage converter

I_{in} represents the current generated by a certain transducer. If we assume that the op amp is ideal, KCL at node *N1* gives

$$I_1 + \left(\frac{V_{out} - 0}{R} \right) = 0 \Rightarrow \boxed{V_{out} = -RI_1} \tag{1.16}$$

The “gain” of this amplifier is given by R. This gain is also called the sensitivity of the converter. Note that if high sensitivity is required for example 1V/μV then the resistance R should be 1 MΩ. For higher sensitivities unrealistically large resistances are required.

A current to voltage converter with high sensitivity may be constructed by employing the T feedback network topology shown on Figure 6.

In this case the relationship between *Vout* and I_1 is

$$V_{out} = -\left(1 + \frac{R2}{R1} + \frac{R2}{R} \right) I_1 \tag{1.17}$$

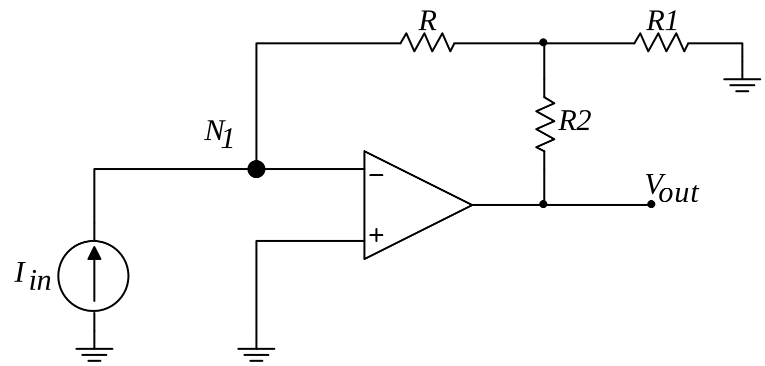


Figure 6. Current to voltage converter with T network

Voltage to Current converter

A voltage to current (V-I) converter accepts as an input a voltage Vin and gives an output current of a certain value.

In general the relationship between the input voltage and the output current is

$$I_{out} = SV_{in} \tag{1.18}$$

Where S is the sensitivity or gain of the V-I converter.

Figure 7 shows a voltage to current converter using an op-amp and a transistor. The op-amp forces its positive and negative inputs to be equal; hence, the voltage at the negative input of the op-amp is equal to *Vin*. The current through the load resistor, RL, the transistor and R is consequently equal to *Vin/R*. We put a transistor at the output of the op-amp since the transistor is a high current gain stage (often a typical op-amp has a fairly small output current limit).

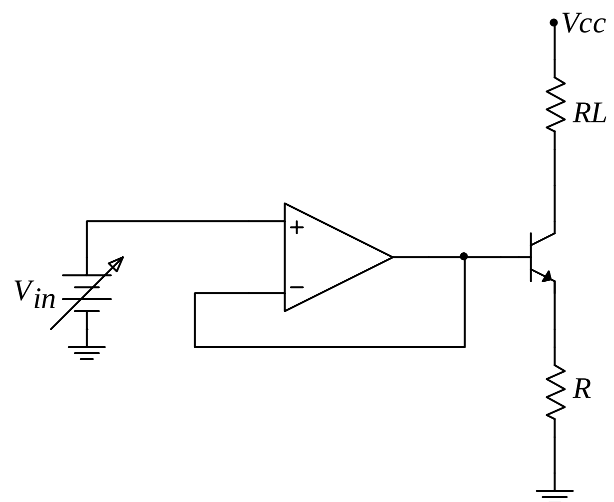


Figure 7. Voltage to current converter

Amplifiers with reactive elements

We have seen that op amps can be used with negative feedback to make simple linear signal processors. Examples include amplifiers, buffers, adders, subtractors, and for each of these the DC behavior described the apparent behavior over all frequencies. This of course is a simplification to treat the op amp ideally, as through it does not contain any reactive elements. Providing we keep the operating conditions out of the slew rate limit then this is a reasonable model. Here we wish to extend this picture of op amp operation to include circuits that are designed to be frequency dependent. This will enable the construction of active filters, integrators, differentiators and oscillators.

The feedback network of an op-amp circuit may contain, besides the resistors considered so far, other passive elements. Capacitors and inductors as well as solid state devices such as diodes, BJTs and MOSFETs may be part of the feedback network.

In the general case the resistors that make up the feedback path may be replaced by generalized elements with impedance *Z1* and *Z2* as shown on Figure 8 for an inverting amplifier.

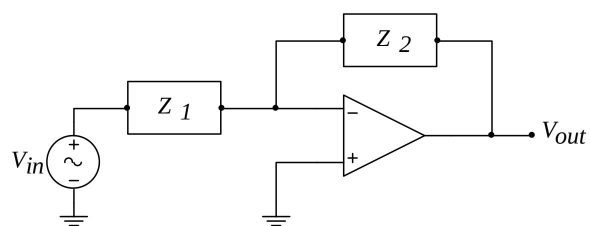


Figure 8. Inverting amplifier with general impedance blocks in the feedback path.

For an ideal op-amp, the transfer function relating *Vout* to *Vin* is given by

$$\frac{V_{out}}{V_{in}} = -\frac{Z_2(\omega)}{Z_1(\omega)} \tag{1.19}$$

Now, the gain of the amplifier is a function of signal frequency (ω) and so the analysis is to be performed in the frequency domain. This frequency dependent feedback results in some very powerful and useful building blocks.

The Integrator: Active Low Pass Filter

The fundamental integrator circuit (Figure 9) is constructed by placing a capacitor C, in the feedback loop of an inverting amplifier.

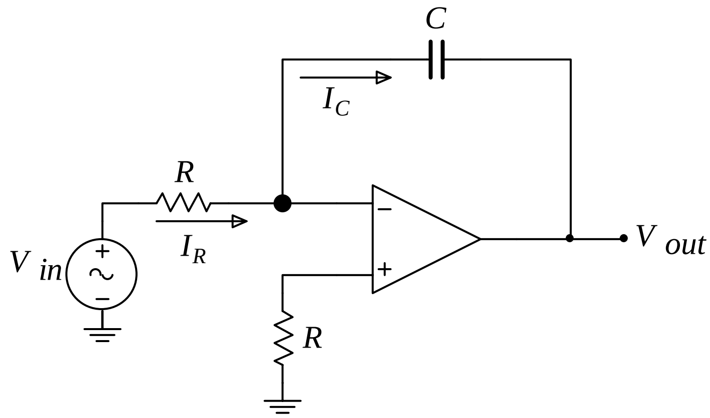


Figure 9. The integrator circuit

Assuming an ideal op-amp, current conservation at the indicated node gives

$$\begin{aligned} I_R &= I_C \\ \frac{V_{in}}{R} &= -C\frac{dV_{out}}{dt} \end{aligned} \tag{1.20}$$

Rearranging Equation (1.20) and integrating from 0 to t, we obtain

$$\int dV_{out} = -\int \frac{V_{in}(\tau)}{RC}d\tau \quad \Rightarrow \quad V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau + V_{out}(0) \tag{1.21}$$

The output voltage is thus the integral of the input. The voltage $V_{out}(0)$ is the constant of integration and corresponds to the capacitor voltage at time t = 0.

The frequency domain analysis is obtained by expressing the impedance of the feedback components in the complex plane. The transfer function may thus be written as

$$\frac{V_{out}}{V_{in}} = -\frac{Z_C}{Z_R} = \frac{\frac{1}{j\omega C}}{R} = \frac{j}{\omega RC} \tag{1.22}$$

The above expression indicates that there is a 90° phase difference between the input and the output signals. This 90° phase shift occurs at all frequencies. The gain of the amplifier given by the modulus $\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\omega RC}$ is also a function of frequency. For dc signals with ω=0 the gain is infinite and it falls at a rate of 20dB per decade of frequency change. The infinite gain for dc signals represents a practical problem for the circuit configuration of Figure 27. Since the equivalent circuit of a capacitor for ω=0 is an open circuit, the feedback path is open. This lack of feedback results in a drift (cumulative summing) of the output voltage due to the presence of small dc offset voltages at the input. This problem may be overcome by connecting a resistor, R_F, in parallel with the feedback capacitor C as shown on Figure 10.

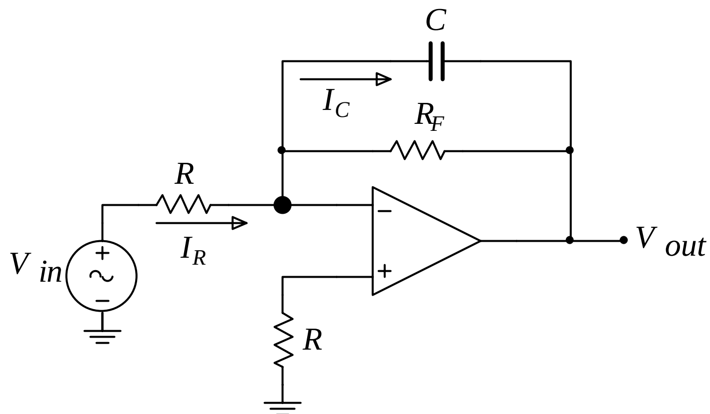


Figure 10. Active Low Pass filter

The feedback path consists of the capacitor C in parallel with the resistor RF. The equivalent impedance of the feedback path is

$$Z_F = \frac{R_F Z_C}{R_F + Z_C} = \frac{\dfrac{R_F}{j\omega C}}{R_F + \dfrac{1}{j\omega C}} = \frac{R_F}{1 + j\omega R_F C} \tag{1.23}$$

The transfer function $\frac{V_{out}}{V_{in}} = -\frac{Z_F(\omega)}{Z_1(\omega)}$ becomes

$$\frac{V_{out}}{V_{in}} = -\frac{Z_F(\omega)}{Z_1(\omega)} = -\frac{R_F}{R_1}\frac{1}{1 + j\omega R_F C} = -\frac{R_F}{R_1}\frac{1}{1 + \dfrac{j\omega}{\omega_{\mathrm{H}}}} \tag{1.24}$$

Where

$$\omega_H \equiv \frac{1}{R_F C} \tag{1.25}$$

Figure 11 shows the logarithmic plot of $\left|\frac{V_{out}}{V_{in}}\right|$ versus frequency. At frequencies much less than ω_H ($\omega \ll \omega_H$) the voltage gain becomes equal to $\frac{R_F}{R}$, while at frequencies higher than ω_H ($\omega \gg \omega_H$) the gain decreases at a rate of 20dB per decade.

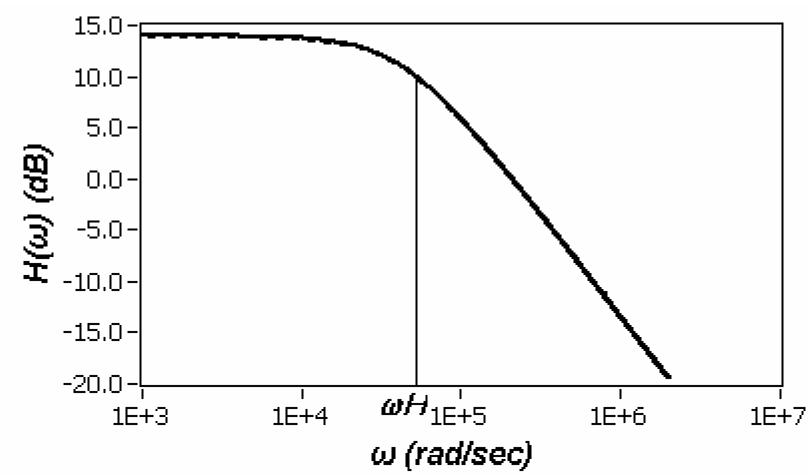


Figure 11. Bode plot of active low pass filter with a gain of 5.

So we have seen that the integration is achieved by charging the feedback capacitor. For an integrator to be useful it must be allowed to be reset to zero. Since the output is stored in the charge of the feedback capacitor, all we need to do is to short out the capacitor in order to reset the integrator.

Integrators are very sensitive to DC drift, small offsets lead to a steady accumulation of charge in the capacitor until the op amp output saturates. We can avoid this by providing another feedback path for DC. The circuit incorporates a shorting path across the capacitor as shown on Figure 12.

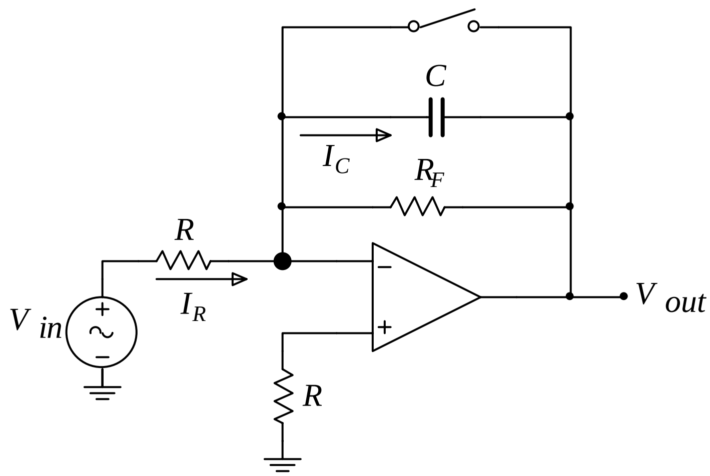


Figure 12. Integrator with reset button

The Differentiator: Active High Pass Filter

A differentiator circuit may be obtained by replacing the capacitor with an inductor for Figure 9. In practice this is rarely done since inductors are expensive, bulky and inefficient devices. Figure 13 shows a fundamental differentiator circuit constructed with a capacitor and a resistor.

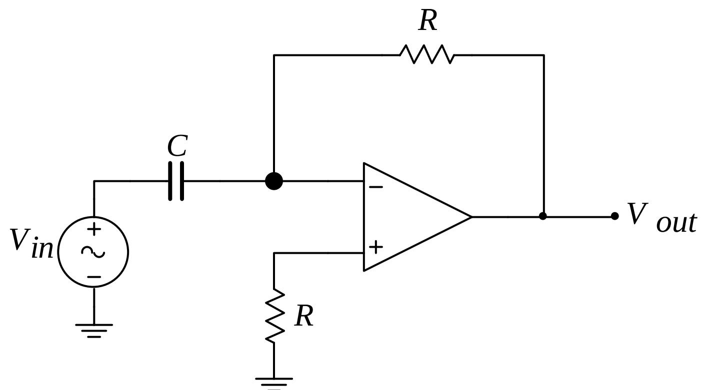


Figure 13. The differentiator circuit

For an ideal op-amp, the current flowing through the capacitor, $C\frac{dV_{in}}{dt}$, is equal to the current flowing through the resistor, $\frac{V_{out}}{R}$, and thus

$$V_{out} = -RC\frac{dV_{in}}{dt} \tag{1.26}$$

The output is thus proportional to the derivative of the input.

As the integrator is sensitive to DC drifts, the differentiator is sensitive to high frequency noise. The differentiator thus is a great way to search for transients, but will add noise. The integrator will decrease noise. Both of these arguments assume the common situation of the noise being at higher frequency than the signal.

Active Band Reject Filter

The integrator and differentiator demonstrate that op amp circuits can be designed to be frequency dependent. This permits the design of active filters, a filter that has gain. We saw before that we could design passive filters based on LC circuits, active filters are no more complicated. Simple selective filters can be made through a frequency dependent impedance in the feedback loop.

Consider the band reject circuit shown on Figure 14.

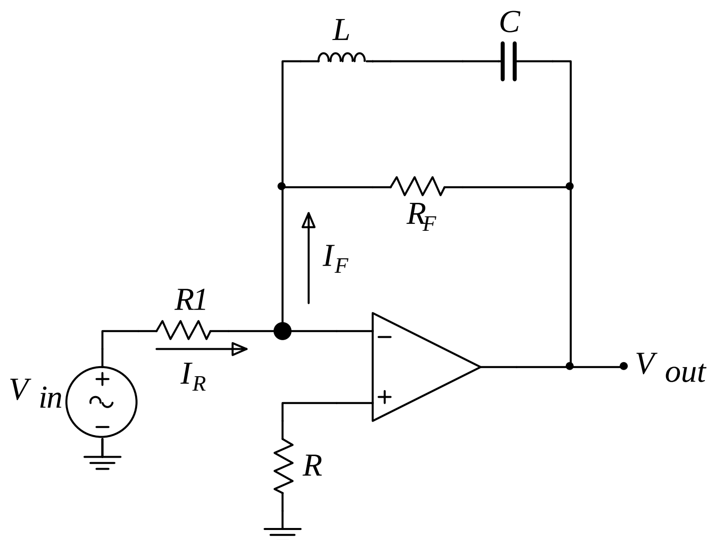


Figure 14. Active band reject filter

We can understand how this circuit works without any detailed calculations. All we need to do is look at the feedback loops. There are two paths in the feedback loop: a frequency independent path with resistance R_F, and a frequency dependent path with impedance given by

$$Z_{\omega} = j\left(\omega L - \frac{1}{\omega C}\right) \tag{1.27}$$

Let's look at the behavior of the circuit as a function of frequency.

For DC signals ($\omega = 0$) the capacitor acts as an open circuit and the equivalent circuit is shown on Figure 15.

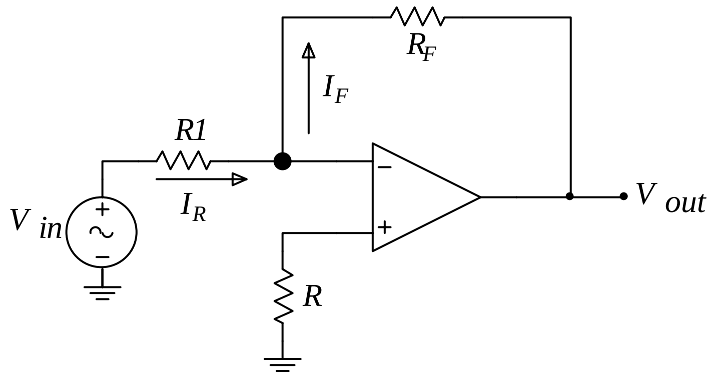


Figure 15

Similarly at high frequencies ($\omega \to \infty$) the inductor acts as an open circuit and the equivalent circuit is the same as the one shown on Figure 15.

Therefore the voltage transfer characteristics at DC and at high frequency are the same with a gain given by

$$G = \frac{V_{out}}{V_{in}} = -\frac{R_F}{R_1} \tag{1.28}$$

The other frequency of interest if the resonance frequency, which occurs when Z_ω as given by Equation (1.27) is equal to zero. The resonance frequency is

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{1.29}$$

and the circuit reduces to the one shown on Figure 16.

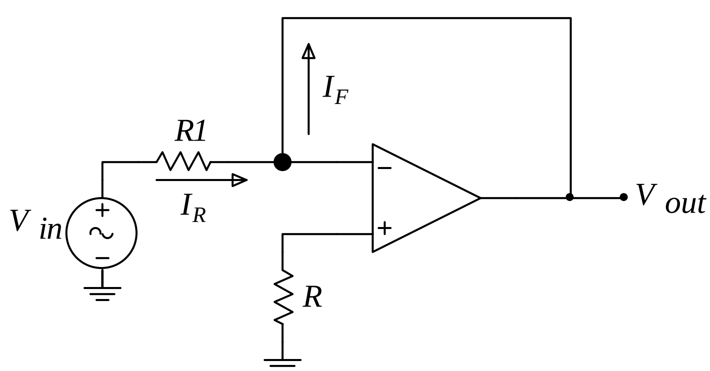


Figure 16

which gives *Vout=0* at $\omega = \omega_o$.

So this is a filter that passes and amplifies every frequency except the resonance frequency.

For the full analysis of this active filter we may write down the complete expression for the impedance of the feedback loop which is

$$Z_F = Z_\omega // R_F = \frac{j\left(\omega L - \frac{1}{\omega C}\right)R_F}{R_F + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{j\left(\omega^2 LC - 1\right)R_F}{\omega R_F C + j\left(\omega^2 LC - 1\right)} \tag{1.30}$$

And thus the general transfer function of the filter is

$$\frac{V_{out}}{V_{in}} = -\frac{Z_F}{R_1} = \frac{j\left(\omega^2 LC - 1\right)R_F}{R_1\left[\omega R_F C + j\left(\omega^2 LC - 1\right)\right]} \tag{1.31}$$

Diodes and transistors in op-amp circuits.

Diodes and transistors may also be used in op-amp circuits. The nonlinear behavior of these devices result in very interesting and useful non-linear op-amp circuits.

Logarithmic Amplifier

If we are interested in processing a signal that has a very wide dynamic range we take advantage of the exponential i-v characteristics of the diode and design an amplifier whose output is proportional to the logarithm of the input.

In practice we may have the voltage signal that corresponds to a certain chemical activity such as the activity, or concentration, of hydrogen ions in a solution which represents the pH of the solution. In this case the voltage is exponentially related to the concentration (pH).

$$Vi = Vo\,k\ln(pH) \tag{1.32}$$

If we use this signal as the input to an inverting amplifier we may linearize the signal by using a diode in the feedback path of the amplifier.

Recall the i-v relationship for a diode

$$\begin{aligned} I &= Io\left[e^{qV/kT} - 1\right] \\ &\simeq Ioe^{qV/kT} \end{aligned} \tag{1.33}$$

Consider the circuit shown on Figure 17.

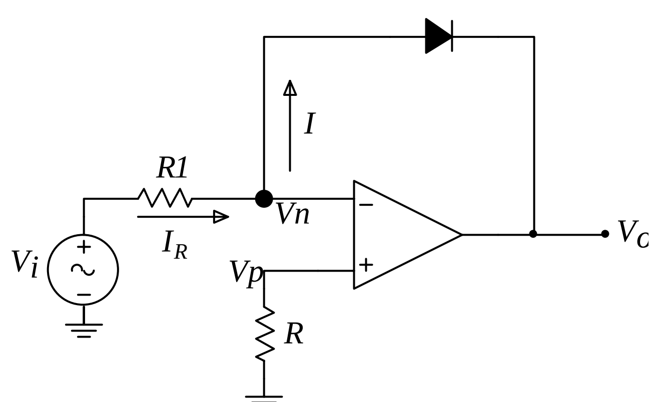


Figure 17. Logarithmic Amplifier

KCL at the indicated node gives

$$\frac{Vi - Vn}{R1} = Ioe^{q(Vn - Vo)/kT} \tag{1.34}$$

And since $Vn = Vp = 0$ we obtain

$$\frac{Vi}{R1} = Ioe^{q(-Vo)/kT} \tag{1.35}$$

And solving for *Vo* gives the desired relationship.

$$\begin{aligned} Vo &= -\frac{kT}{q}\ln\frac{Vi}{Io\,R1} \\ Vo &= -\underbrace{\frac{kT}{q}}_{a}\ln(Vi) + \underbrace{\frac{kT}{q}\ln(IoR1)}_{b} \\ Vo &= \boxed{-a\ln(Vi) + b} \end{aligned} \tag{1.36}$$

Similarly and **antilogarithmic amplifier** may be constructed by placing the diode in series with the signal source as shown on Figure 18.

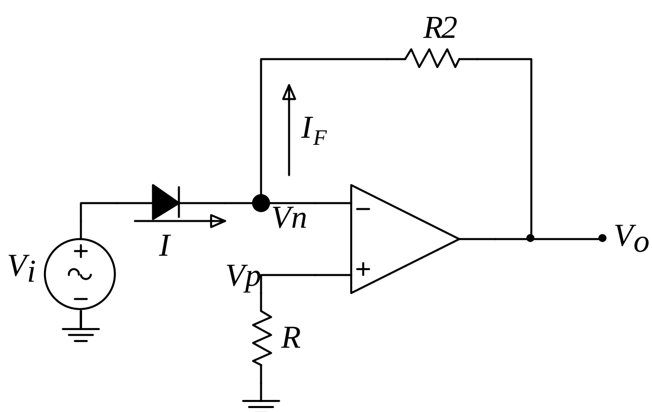


Figure 18. Antilogarithmic amplifier

Here you may show that

$$Vo = -IoR2e^{qVi/kT} \tag{1.37}$$

Superdiode. Precision half wave Rectifier

The diode rectifier circuit and its associated voltage transfer characteristic curve are shown on Figure 19(a) and (b).

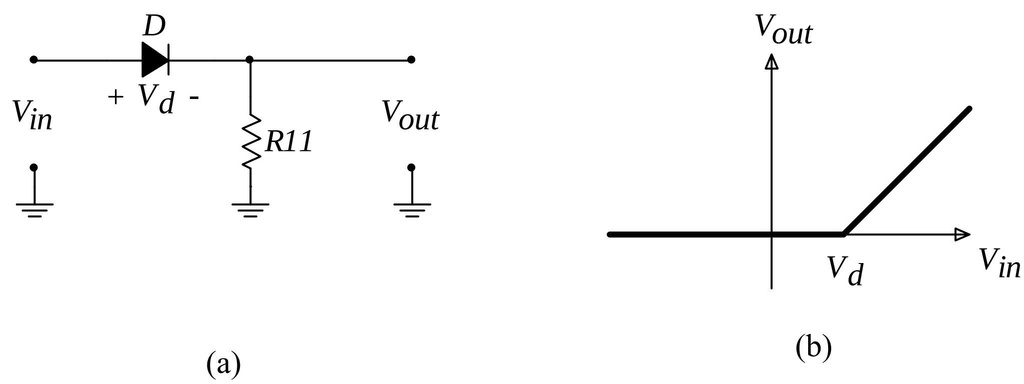


Figure 19. Diode rectifier circuit (a) and voltage transfer curve (b)

The offset voltage *Vd* is about 0.7 Volts and this offset value is unacceptable in many practical applications. The operational amplifier and the diode in the circuit of Figure 20 form an ideal diode, a superdiode, and thus they eliminate the offset voltage Vd from the voltage transfer curve forming an ideal half wave rectifier.

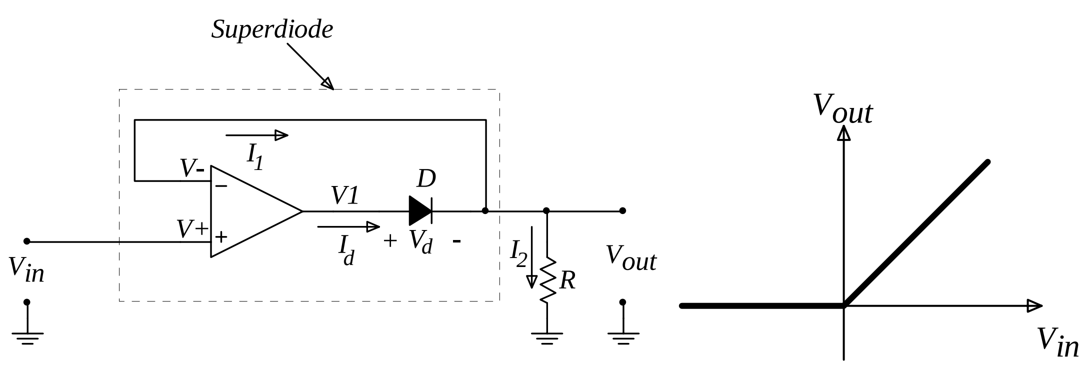


Figure 20. Precision half wave rectifier circuit and its voltage transfer curve.

Let's analyze the circuit by considering the two cases of interest: *Vin>0* and *Vin<0*.

For *Vin<0* the current I_2 and id will be less than zero (point in a opposite direction to the one indicated). However, negative current can not go through the diode and thus the diode is reverse biased and the feedback loop is broken. Therefore the current I_2 is zero and so the output voltage is also zero, *Vout=0*. Since the feedback loop is open the voltage *V1* at the output of the op-amp will saturate at the negative supply voltage.

For $Vin>0$, $Vout=Vin$ and the current $I_2=I_d$ and the diode is forward biased. The feedback loop is closed through the diode. Note that there is still a voltage drop Vd across the diode and so the op-amp output voltage $V1$ is adjusted so that $V1=Vd+Vin$.

Problems

P1. Resistors R1 and R2 of the circuit on Figure P1 represent two strain gages placed across each other on a beam in order to measure the tensile and compressive stains. R1 and R2 vary symmetrically by a factor δ as $R1=R(1-\delta)$ and $R2=R(1+\delta)$.

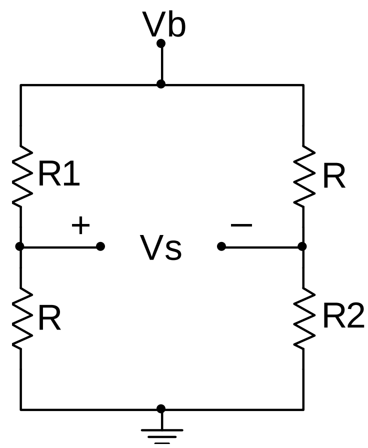


Figure P1.

Design an amplifier so that the output varies from -10V to +10V as the parameter δ varies from -0.01 to +0.01. The bias voltage Vb=+10V and R=10kΩ.

P2. The resistors of the amplifier circuit of Figure P2 have a tolerance of $\pm\delta$%.

1. Assume that Vin is known precisely and calculate the deviation in the output voltage Vout.

2. For R1=15kΩ±5% and R2=200kΩ±5% and Vin=100mV±1% calculate the output voltage.

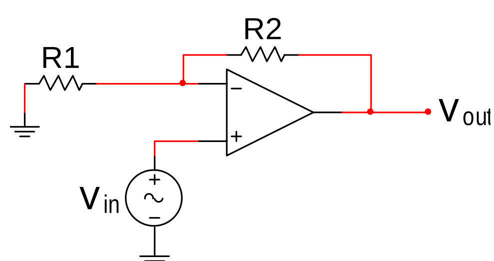
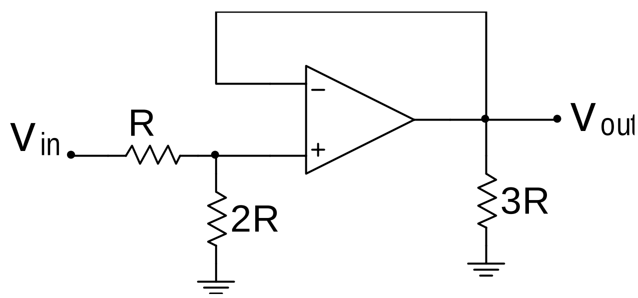
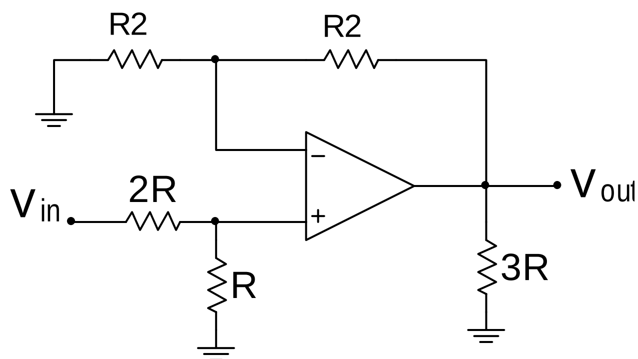


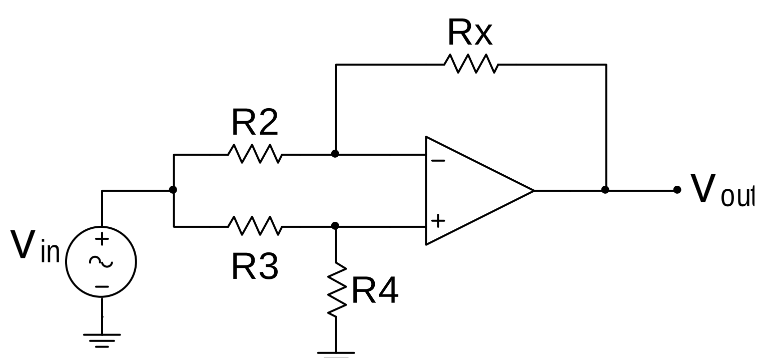
Figure P2.

P3. Calculate the output voltage Vout for the following circuits.





P4 For the circuit on Figure P4 determine the value of resistor Rx so that the output voltage is zero.



P5 The circuit on Figure P5 is a current source.

1. Show that the amount of current delivered to the load is controlled by resistor R3.
2. Calculate the resistance seen by the Load.

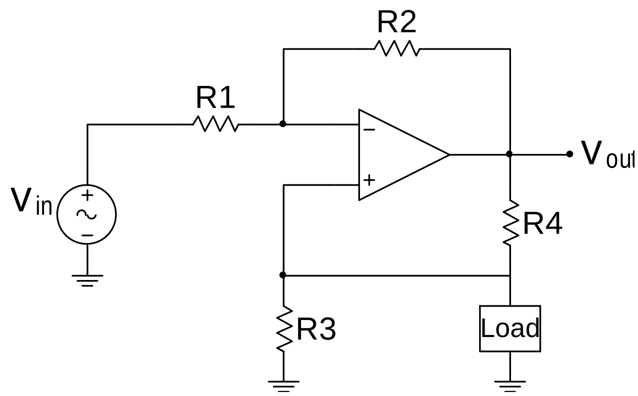


Figure P5

P6. For the circuit on Figure P6 calculate the currents i1, i2, i3, i4 and the voltages v1 and Vout. Refer your answers to the indicated current directions.

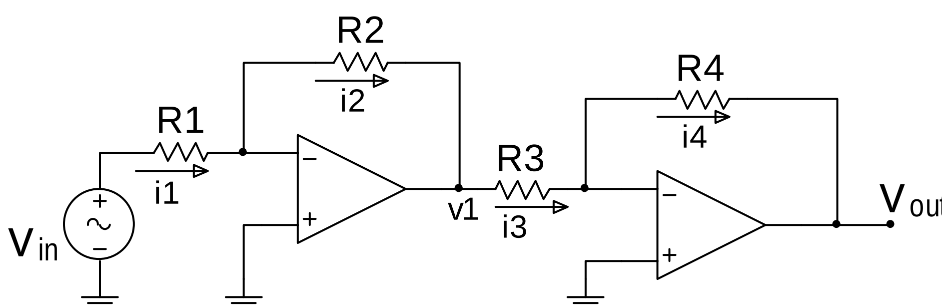
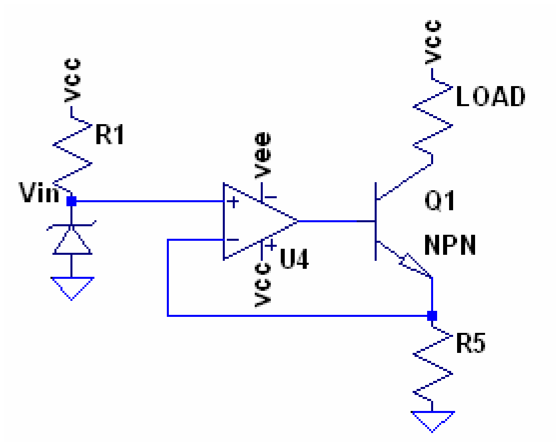


Figure P6

P7



For the voltage to current converter, briefly (in 1-2 sentences) describe what happens when each of the following faults happen (alone, independently of other faults).

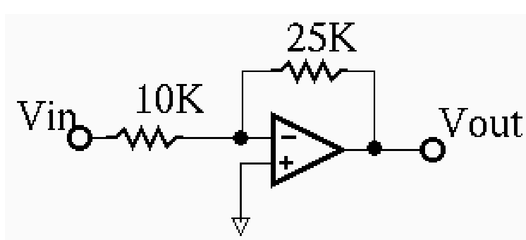
a. the Zener diode is shorted

b. R5 has a bad soldering joint and is opened

c. The load becomes shorted

d. The connection between the op-amp output and the base of the transistors becomes opened

P8: Op-amp nonidealities:



a. What is the effect of a 1uA input bias current on the output voltage of the opamp?

b. What is the effect of a 5mV input offset voltage on the output of the opamp?