

# 14.01 Fall 2010: Midterm 1 Solution Set

## 1. True/False/Uncertain Questions

Write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

- (a) A consumer finds two goods to be perfectly substitutable. Claim: The optimal bundle for this consumer will always be a corner solution.

*False.* Consider for example the utility function  $u = x + y$ . The optimal bundle will be a corner solution if the prices of the two goods are different; if the prices are the same, then any allocation is optimal.

- (b) Innovations in the production of batteries lead to a rightward shift in the market supply for hybrid cars, while demand stays the same. Since this leads to a decrease in the equilibrium price and an increase in the equilibrium quantity, demand is more inelastic at the new equilibrium.

*Uncertain.* This is true in the case of linear demand, since elasticity of demand is increasing in price along a linear demand curve. However, this is false in the case of a constant elasticity demand curve.

- (c) A consumer has selected an optimal bundle of two goods that includes some of each good. The price of one good increases. Claim: her utility is lower after the price increase compared to before it.

*True.* If the consumer was indifferent to one of the goods, she would have selected a bundle of only the other good. Clearly, she obtains utility from both goods; thus when the price of either increases and her budget constraint shifts in, she will be on a lower indifference curve.

- (d) When market demand and supply shift in opposite directions, we can unambiguously say how the equilibrium price and quantity change.

*False.* When demand and supply shift in the opposite direction we can unambiguously say how the equilibrium price changes (equilibrium price increases if supply shifts to the left and demand shifts to the right and vice versa). However, the effect on equilibrium quantity is ambiguous; it could either increase or decrease in both cases.

## 2. Market demand for frozen yogurt

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand curve of  $P = 5 - \frac{1}{2}Q$ . The second type are crazy about frozen yogurt and have an inverse demand curve of  $P = 20 - Q$ . In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

- (a) Using the individual demand curves above, derive the market demand for frozen yogurt in Smallville. Plot the market demand curves.

*The demand of the first group is*

$$Q_1^D = \begin{cases} 10 - 2P & , P \leq 5 \\ 0 & , P > 5 \end{cases}$$

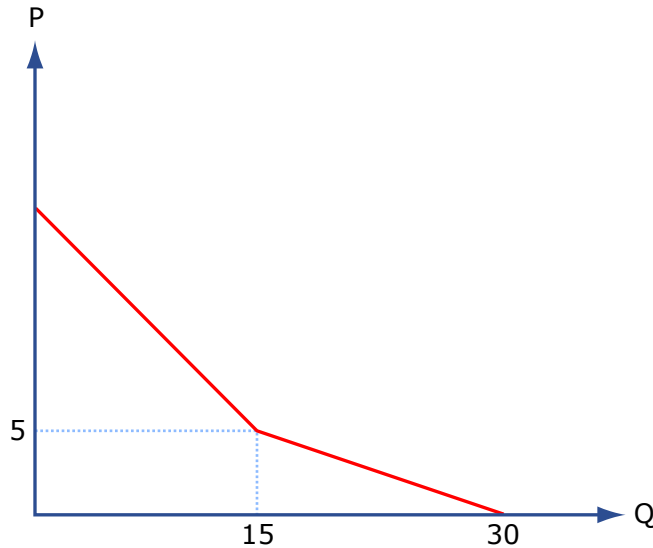


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and the demand of the second group is

$$Q_2^D = 20 - P$$

Summing the two demand curves, we get a market demand curve of

$$Q_M^D = Q_1^D + Q_2^D = \begin{cases} 30 - 3P & , P \leq 5 \\ 20 - P & , P > 5 \end{cases}$$

The inverse market demand curve is then:

$$P = \begin{cases} 20 - Q & Q \leq 15 \\ 10 - \frac{1}{3}Q & Q > 15 \end{cases}$$

- (b) Suppose that the market supply for frozen yogurt in Smallville is given by  $Q^S = 2 + P$ . Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? (Hint: Check the equilibrium price and quantity you get on a graph)

*The market demand curve is piecewise linear. However, since at  $p = 5$   $Q^S = 7 < 15 = Q^D$  it follows that supply would intersect demand to the left of the kink. Hence,  $Q^S = Q^D$  or  $2 + P = 20 - P$  and hence  $P^* = 9$ . This means that  $Q^* = 11$ . Only the crazy consumer buys at the equilibrium price.*

### 3. Consumer preferences and optimal allocations

Mary is starting a jewelry collection. She wants to own matched sets of three bracelets and one necklace that can be worn together, and she doesn't want to own any bracelets or necklaces that are not in a matched set of this size.

- (a) Draw Mary's indifference curves and write her utility function. Put bracelets on the y axis and necklaces on the x axis. Assume she receives utility of 3 utils from each matched jewelry set she owns.

$$U(b, n) = 3 \min(n, \frac{1}{3}b) \text{ or } U(b, n) = 3 \min(3n, b)$$

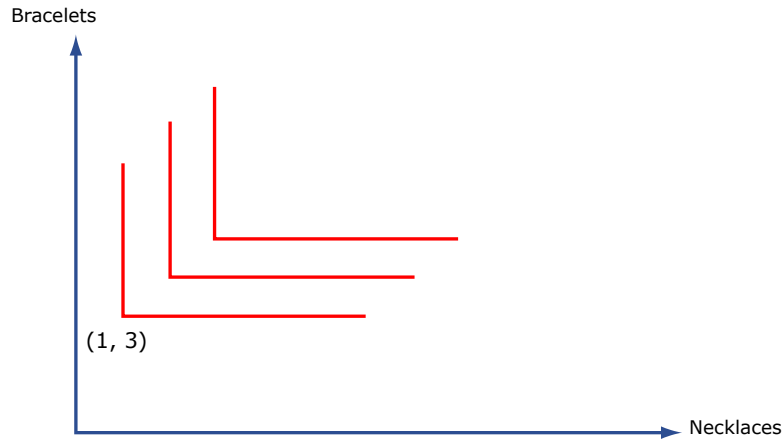


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- (b) Currently, Mary has 32 dollars to spend. The price of necklaces is  $p_n = 2$  and the price of bracelets is  $p_b = 2$ . What is the optimal allocation of necklaces and bracelets for Mary? How much utility does she achieve from this allocation?

*We can find the optimal allocation by finding the intersection of the lines  $b = 3n$  and  $2n + 2b = 32$ . Substituting in to the budget constraint, this yields the following.*

$$\begin{aligned} 2n + 6n &= 32 \\ n &= 3 \end{aligned}$$

*We have  $n = 4$ ,  $b = 12$ ,  $u = 12$ .*

- (c) Due to a shortage of gold, the price of necklaces increases to  $p_n = 10$ . What is the new allocation of necklaces and bracelets at this income level, and what utility does Mary obtain? What is the proportional decrease in her utility?

*Re-solve the problem with a new budget constraint. This yields*

$$10n + 6n = 32$$

*We have  $n = 2$ ,  $b = 6$ ,  $u = 6$ .*

- (d) Luckily, Mary's parents value their daughter's utility, and are willing to give her enough income to ensure that she has the same utility she did prior to the price change. How much extra money do they have to give her?

*Mary's parents have to give her sufficient income to buy two more matched sets at the new prices. This requires the purchase of two necklaces and six bracelets, at a cost of  $2 * 10 + 6 * 2 = 32$ . They have to double her income, increasing it by \$32, in order to maintain her at the same utility level.*

- (e) Mary has a sister Lily who doesn't like wearing matched sets of jewelry and has different preferences. Her utility function is  $nb^2$ . If she started with the same jewelry budget as Mary of 32 dollars and then faced the same price shock, what would be the decrease in her utility when the price of necklaces increases from \$2 to \$10?

*First, solve for Lily's original optimal allocation. We set the ratio of marginal utilities equal to the ratio of prices.*

$$\begin{aligned} \frac{b^2}{2nb} &= \frac{2}{2} \\ b &= 2n \end{aligned}$$

*Substituting into the budget constraint, this yields*

$$2(2n) + 2n = 32$$

This yields  $n = \frac{16}{3}$ ,  $b = \frac{32}{3}$ , and  $u = \frac{16}{3}(\frac{32}{3})^2 = \frac{512}{27}$   
 When prices increase, we now re-solve the problem.

$$\begin{aligned} \frac{b^2}{2nb} &= \frac{10}{2} \\ b &= 10n \end{aligned}$$

Substituting into the budget constraint, this yields

$$2(10n) + 10n = 32$$

This yields  $n = \frac{16}{15}$ ,  $b = \frac{160}{15}$ , and  $u = \frac{16}{15}(\frac{160}{15})^2 = \frac{2560}{225}$ . To calculate the decrease in utility, subtract the second level of utility from the first.

$$\begin{aligned} &= \left( \frac{16 * 16^2 * 4}{27} - \frac{16 * 16^2 * 100}{27 * 5^3} \right) / \left( \frac{16 * 16^2 * 4}{27} \right) \\ &= \frac{16 * 16^2 (4 * 5^3 - 100)}{27 * 5^3} \\ &= 485.4 \end{aligned}$$

Lily experiences a 80% decline in her utility.

- (f) Mary and Lily's parents are going to give a gift of equal monetary value to both sisters. They are trying to decide whether to give cash or give jewelry. Which sister is more likely to prefer cash? Please explain intuitively and/or graphically; there is no need for algebra in this section.

Mary is more likely to prefer cash, because she only receives utility from a gift that is given as a matched set and no utility from any other type of gift. You can portray this graphically by showing a graph of the two utility functions and indicating that for Mary, any increase in the quantity of one good or the other keeps her on the same indifference curve, while for Lily, any increase will move her to a new indifference curve. The below graph shows the result of a gift of necklaces only to both girls. Mary is on the same utility curve, but Lily has increased utility. In this case and others like it, Mary is more likely to prefer cash.

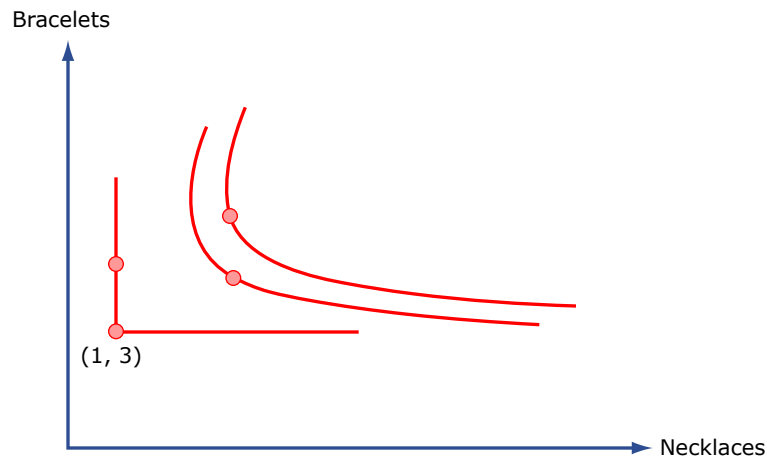


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#### 4. Labor markets and labor supply shocks

Consider the labor market in the country of Widgetland. The demand for labor is given by:

$$L^D = 34 - 4w$$

where  $w$  is the wage rate.

Labor supply is:

$$L^S = \bar{L} + 2w$$

where  $\bar{L}$  is the number of people in the country willing to work at a wage of zero.

- (a) Suppose that  $\bar{L} = 10$ . Find the equilibrium wage and equilibrium demand for labor. Is demand for labor elastic or inelastic at the equilibrium wage?

To find equilibrium wage we set  $L^S = L^D$  and hence  $34 - 4w = 10 + 2w$ . Hence,  $w^* = 4$ .  $L^* = 18$ . The elasticity of demand at the equilibrium wage is

$$\epsilon_D = \frac{dL^D}{dw} \frac{w^*}{L^*} = -4 \frac{4}{18} = -\frac{16}{18} = -\frac{8}{9}$$

Hence, demand for labor is inelastic at  $w^*$ .

Suppose that there is a sudden influx of migrant labor, which increases the number of people willing to work at a wage of zero to  $\bar{L} = 16$ . For the remainder of the problem, set  $\bar{L} = 16$ .

- (b) Compute the new market equilibrium. What happens to the equilibrium wage rate?

In the new equilibrium  $34 - 4w = 16 + 2w$ . Hence,  $w^* = 3$ .  $L^* = 22$ . Hence the equilibrium wage rate decreases.

- (c) In reality an increase in population should affect the demand for labor as well as the supply. Explain how the equilibrium wage and labor demanded will change compared to the market equilibrium in part (a) if demand for labor were to increase as well.

A shift in demand in the same direction as the shift in supply increases the equilibrium demand for labor unambiguously. However, the change in the wage rate is ambiguous as the wage rate could either increase or decrease.

- (d) The government in Widgetland becomes worried about the upcoming election and decides to appease voters by imposing a minimum wage of  $\underline{w} = 4$ . What happens in the labor market as a result? What is the demand and supply for labor now? Include a graph in your explanation.

Since  $\underline{w} > w^*$  it follows that the minimum wage will be binding. At the minimum wage demand for labor is  $L^D = 18$  whereas supply for labor is  $L^S = 24$ . Hence, as a result of the minimum wage there is unemployment of  $L^S - L^D = 6$ .

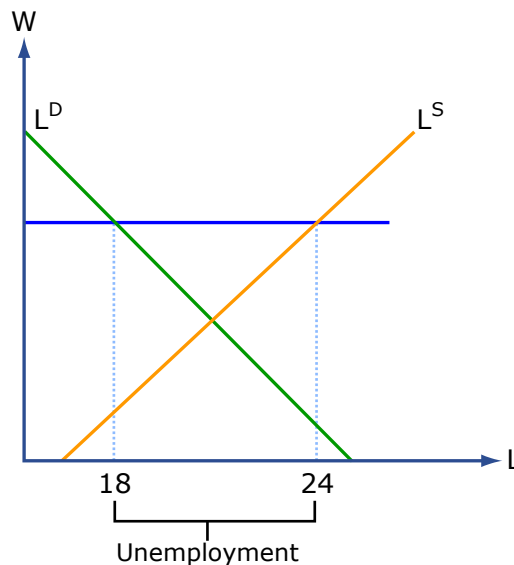


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- (e) The government is unhappy with the results of the minimum wage law and repeals it. Instead it introduces a subsidy of  $\tau = 1$  dollar on labor that is paid to workers. What happens to the equilibrium wage and labor used as a result of this subsidy? How much do workers get in equilibrium?

*A subsidy behaves like a negative tax. Hence, it effectively shifts supply down by  $\tau$ . The new equilibrium wage is determined where labor supply with the subsidy equals labor demand, i.e.  $L^S(p + \tau) = L^D$  or  $16 + 2(w + \tau) = 34 - 4w$ . Hence,  $w^* = \frac{16}{6} = \frac{8}{3}$ . Hence workers get paid  $w^* + \tau = \frac{8}{3} + 1 = \frac{11}{3}$ .*

## 5. Income and substitution effects

Glenn's utility function for goods  $X$  and  $Y$  is represented as  $U(X, Y) = X^{0.2}Y^{0.8}$ . Assume his income is \$100 and the prices of  $X$  and  $Y$  are \$10 and \$20, respectively.

- (a) Express his marginal rate of substitution (MRS) between goods  $X$  and  $Y$ . As the amount of  $X$  increases relative to the amount of  $Y$  along the same indifference curve, does the MRS increase or decrease? Explain.

$$MU_X = 0.2(Y/X)^{0.8} \text{ and } MU_Y = 0.8(X/Y)^{0.2}$$

$$MRS_{Y \text{ for } X} = -MU_X/MU_Y = -Y/4X$$

*The MRS (in absolute value) gets smaller as the amount of  $X$  increases relative to  $Y$ . In other words, the more  $X$  (and less  $Y$ ) one has, the less of  $Y$  one is willing to give up in order to obtain an additional unit of  $X$ .*

- (b) What is his optimal consumption bundle  $(X^*, Y^*)$ , given income and prices of the two goods?  
*At the optimal consumption bundle, the MRS is equal to the ratio of prices. That is,  $MRS_{Y \text{ for } X} = -P_X/P_Y$ . Plugging in our prices and the results from (a),*

$$-Y/4X = -10/20 \implies -Y/4X = -1/2 \implies 2Y = 4X \implies Y = 2X$$

*The budget constraint must hold as well:  $I = P_X X + P_Y Y \implies 100 = 10X + 20Y$   
 We now have two equations and two unknowns. Solve for  $(X^*, Y^*)$ :*

$$100 = 10X + 20(2X) \implies 100 = 50X \implies X^* = 2$$

$$Y = 2X = 2(2) \implies Y^* = 4$$

- (c) How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles?

*The utility function does not change and therefore the formula for MRS does not change. Prices change, but the ratio  $-P_X/P_Y$  does not. Therefore, from  $MRS_{Y \text{ for } X} = -P_X/P_Y$  we still get  $Y = 2X$ . The budget constraint changes to  $100 = 20X + 40Y$ . Solving as before (with two equations, two unknowns), we get  $(X^*, Y^*) = (1, 2)$ . If all prices and income change proportionally, the optimal bundle does not change.*

- (d) Derive the demand curve for good  $X$  and demand curve for good  $Y$ .  
*Solve for  $X^*$  and  $Y^*$  as before, except this time do not plug in explicit values for  $P_X, P_Y$  and  $I$ .*

$$MRS_{Y \text{ for } X} = -P_X/P_Y \implies -Y/4X = -P_X/P_Y \implies P_Y Y = 4P_X X$$

*Budget constraint:  $I = P_X X + P_Y Y$   
 Now let's solve for  $X^*$  by substituting out  $Y$ .*

$$I = P_X X + 4P_X X = 5P_X X \implies X^*(P_X; I) = I/(5P_X)$$

*Holding income constant at \$100 gives us a demand curve of  $X^*(P_X; I = 100) = 20/P_X$*

We can solve for  $Y^*$  by substituting out  $X$ :

$$P_Y Y = 4P_X X \implies Y = (4P_X X)/P_Y \implies Y = (4P_X [I/(5P_X)])/P_Y$$

$$Y^*(P_Y; I) = 4I/(5P_Y)$$

Holding income constant at \$100 gives us a demand curve of  $Y^*(P_Y; I = 100) = 80/P_Y$

Now a government subsidy program lowers the price of  $Y$  from \$20 per unit to \$10 per unit.

- (e) Calculate and graphically show the change in good  $Y$  consumption resulting from the program.

To calculate the new bundle we could go through the same procedure as in part (b) or simply use the demand equations derived in part (d).  $X^*$  does not depend on  $P_Y$  so  $X^{*'} = 2$ .

$Y^*$  is a function of its own price and, using demand from (d),  $Y^{*'} = 80/P_Y = 80/10 = 8$ . This change is depicted in the figure below.

- (f) In a clearly labeled diagram with  $Y$  on the y-axis and  $X$  on the x-axis, graphically show the change in consumption attributable to the separate income and substitution effects. No calculations were required for this part.

Refer to the figure below for the separate income and substitution effects. Note that the substitution effect for good  $Y$  may be found by sketching a  $BC_{sub}$  with the new price ratio (parallel to  $BC_1$ ) but tangent to the original indifference curve  $IC_0$ . The corresponding point tells us what a person facing the new price ratio would purchase if utility were somehow held constant at its original level. The income effect brings us from this substitution point to  $Y^{*'}$ .

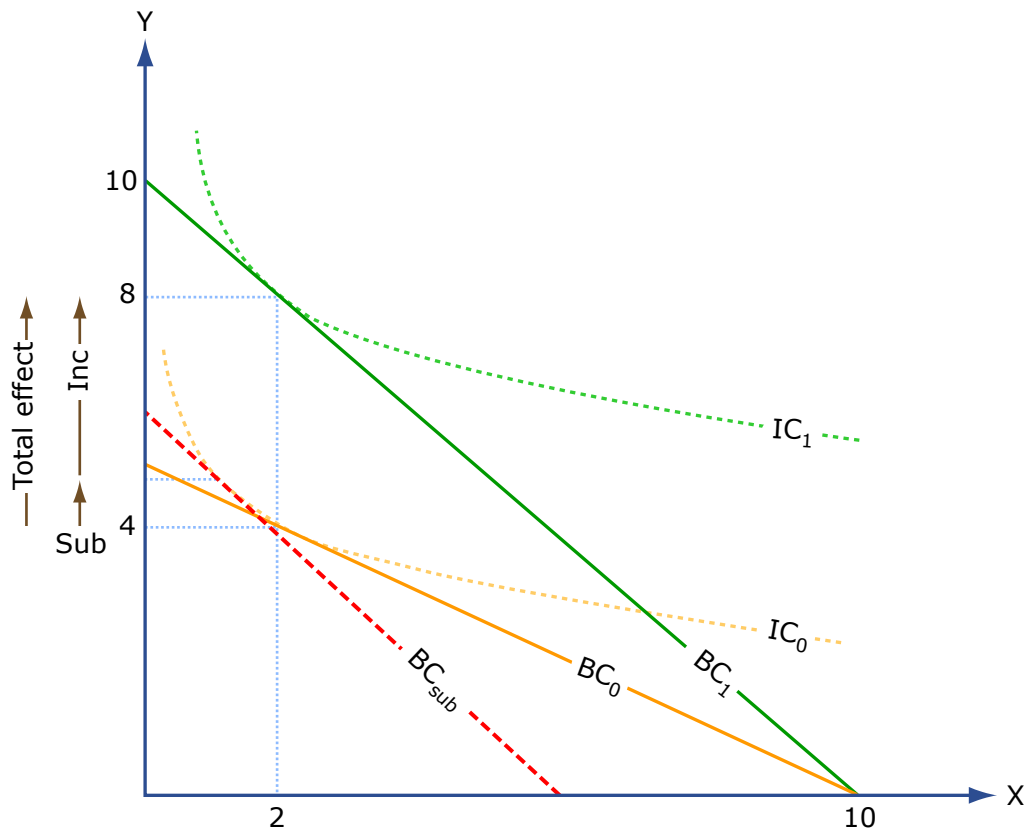


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- (g) How much does the program cost the government?

For each unit of  $Y$  purchased, the government pays out \$10. Since 8 units of  $Y$  are currently purchased, the cost will be  $8(\$10) = \$80$ .

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