18.03 Hour Exam I

March 1, 2006

Your Name
Your Recitation Leader's Name
Your Recitation Time

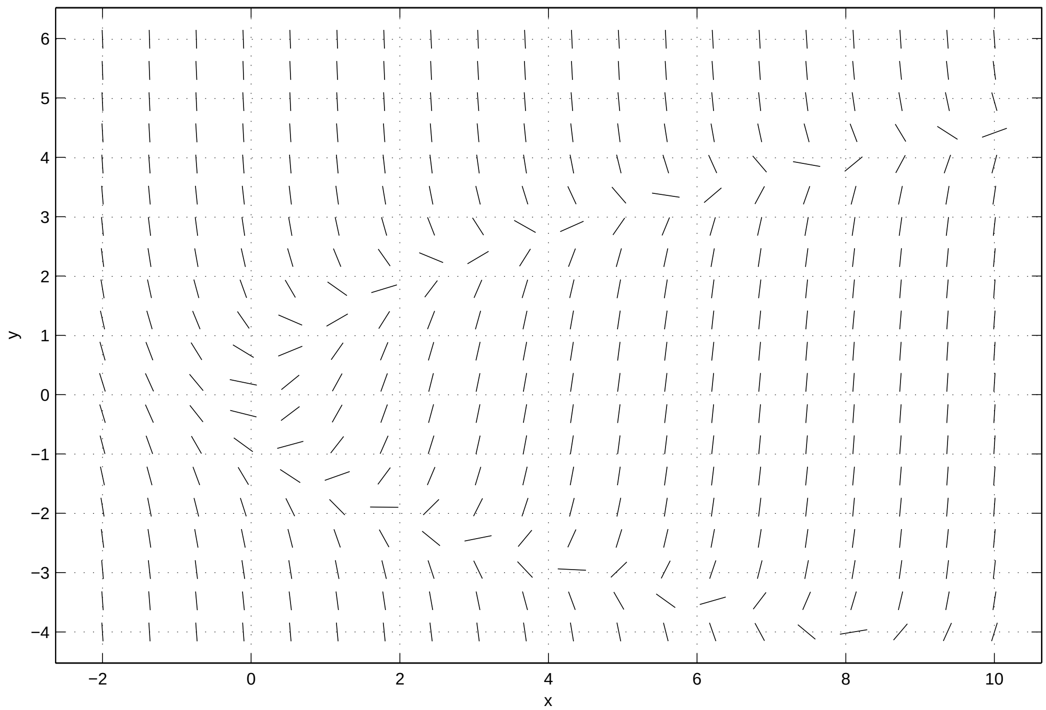
There are six problems. Do all your work on these pages. No calculators or notes may be used. The point value (out of 100) of each problem is marked in the margin. Solutions will be available from the UMO and on the web after 4:00 today, and at recitation. Exams will be handed back at recitation on Tuesday.

Problem	Points
1	
2	
3	
4	
5	
6	
Total	

[12] **1.** **(a)** A pot of water cools at a rate proportional to the difference T between its temperature and room temperature. It is observed that over an interval of $10 \ln 2 \simeq 6.93$ minutes the value of T is halved. On the stove, though, there is a second process at work: heat is added to the water at a constant rate, increasing temperature at the rate of 8 degrees per minute. Write down a differential equation for T which controls it while the pot is on the stove. Justify any constants that appear in the equation.

[12] **(b)** Estimate $y(2)$ where y is the solution of the differential equation $y' = x + y$ with $y(0) = 0$, using Euler's method with step size 1. Do you expect your estimate to be too high or too low? Why?

2. The direction field of a differential equation $y' = F(x, y)$ is illustrated.



[6] **(a)** On the direction field, sketch the graph of the solution $y(x)$ with $y(0) = 0$. Continue it in both directions till it leaves the direction field box.

[4] **(b)** Some solutions of this differential equation grow when x is large and some do not. On the graph, sketch the curve representing the boundary between these two behaviors. Continue it in both directions till it leaves the direction field box.

[4] **(c)** The null-cline is given by the equation $2x = y^2$ Estimate the value $y(50)$ of the solution you graphed in **(a)**.

Is your estimate high or low?

[12] **3.** Find a solution of $\dot{x} = x + 2te^t$, by any method.

[12] 4. Find a sinusoidal solution to the differential equation $\dot{x} - 2x = 4\cos(3t)$. Express your answer as a sum of sines and cosines. You may use any method to find this solution.

[6] **5. (a)** Compute explicitly in the form $a + bi$ all the cube roots of i.

(b)–(e) relate to the sinusoidal function $2\sin(\pi t) - 2\cos(\pi t)$. Determine

[3] **(b)** its period P

[3] **(c)** its amplitude A

[3] **(d)** its phase lag ϕ

[3] **(e)** its time lag t_0

6. A grower of mushrooms wants to maximize his harvest rate a (in tons per week). The number of tons of mushrooms in his farm obeys the logistic equation $\dot{y} = -y^2 + 4y$ in the absence of harvesting.

[8] **(a)** Sketch the phase portrait of this differential equation as it is, in the absence of harvesting. Sketch some solutions.

[6] **(b)** What is the largest harvesting rate a that will allow for a constant mass of mushrooms?

[6] **(c)** The farmer maintains a 3 ton per week harvest rate over a long period of time, with a constant mushroom mass. What must that mass be, approximately?