Recitation 2, February 9, 2006

Direction fields, integral curves, isoclines

An isocline of the differential equation $\frac{dy}{dx} = F(x, y)$ is the portion of the plane where the slope $F(x, y)$ is a constant. A good way to create direction fields is to plot a few isoclines (especially the "null-cline," where $F(x, y) = 0$, and the "infinity-cline," where $F(x, y) = \pm\infty$).

As an example, take the ODE $y' = x - 2y$.

1. Draw a big axis system and plot some isoclines, especially the nullcline. Plot a few solutions.

2. One of the integral curves seems to be a straight line. Is this true? What straight line is it? (i.e. for what m and b is $y = mx + b$ a solution?)

3. As a general thing, if a straight line is an integral curve, how is it related to the isoclines of the equation? What happens in our example?

4. It seems that all the solutions become asymptotic as $x \to \infty$. Explain at least why solutions get trapped between parallel lines of some fixed slope .

5. What can be said in general about when a solution has a critical point? Where are the critical points of the solutions in our example? How many critical points can a single solution have? Can you predict on the basis of an initial value whether or not a solution will have a critical point? When there is one, is it a minimum or a maximum?

6. In lecture the equation $y' = y^2 - x$ was discussed. There is more to say about that example than there was time to describe. Sketch some isoclines and some solutions. One question is: where are the critical points of solutions? Can a solution have more than one?

7. How about points of inflection (where $y'' = 0$)? Hint: differentiate the ODE and then replace y' with the right hand side of the original ODE. (You may want to think about what happens in the $y' = x - 2y$ example as well.)

8. A "separatrix" is a solution such that solutions on one side of it have a fate entirely different from solutions on the other side. The equation $y' = y^2 - x$ exhibits a separatrix. Sketch it and describe the differing behaviors.