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18.02 Multivariable Calculus  
Fall 2007

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## 18.02 Problem Set 3

Due Thursday 9/27/07, 12:45 pm.

### Part A (10 points)

Hand in the underlined problems only; the others are for more practice.

Lecture 7. Thu Sept. 20      Review

Lecture 8. Fri Sept. 21      **EXAM 1**

Lecture 9. Tue Sept. 25      Functions of several variables, Level curves. Partial derivatives.  
Tangent plane approximation.

Read: 13.2, 13.4; Notes TA

Work: 13.2/7, 15, 53-58; 2A/1abcde, 2abcde, 3abc, 5ab; 2B/1ab, 3, 6, 9; 13.4/57.

### Part B (19 points)

**Directions:** Attempt to solve *each part* of each problem yourself. If you collaborate, solutions must be written up independently. It is illegal to consult materials from previous semesters. With each problem is the day it can be done.

Write the names of all the people you consulted or with whom you collaborated and the resources you used.

**Problem 1.** (immediately, 8 points: 2+1+1+2+2)

In 3D computer graphics, one needs to represent 3-dimensional objects on a plane screen, by drawing a given point  $P$  at the place where the line from  $P$  to the eye meets the screen. Suppose that the screen is the  $yz$ -plane, and the eye is at  $E : (2, 0, 0)$ .

a) At what point  $Q : (y, z)$  in the  $yz$ -plane should one represent the point  $P : (x_0, y_0, z_0)$ ? (Express  $y$  and  $z$  in terms of the coordinates of  $P$ . Assume that  $x_0 < 2$ . Why is this assumption legitimate?)

b) What does the image on the screen of a line segment in space look like? (Justify your answer.)

c) A line segment connects  $P_0 : (-1, -3, 1)$  to  $P_1 : (-2, 4, 6)$ . What is drawn on the screen?

d) A bird leaves from  $P_0$  at time  $t = 0$ , and flies in a straight line at constant speed in such a way that it passes through  $P_1$  at time  $t = 1$ . What does the trajectory of the bird (for  $t \geq 0$ ) look like on the screen? Show that, as  $t$  tends to infinity, the trajectory on the screen tends to a limit point (the “vanishing point”), and give its coordinates.

e) In fact, part of the trajectory of the bird is hidden by a vertical fence erected in front of the observer. The fence lies in the plane  $x = 1$ , and its top is at the altitude  $z = 1$ . What portion of the trajectory is hidden?

*Hint:* observe that the points hidden by the fence are exactly those which lie below a certain plane passing through  $E$ .

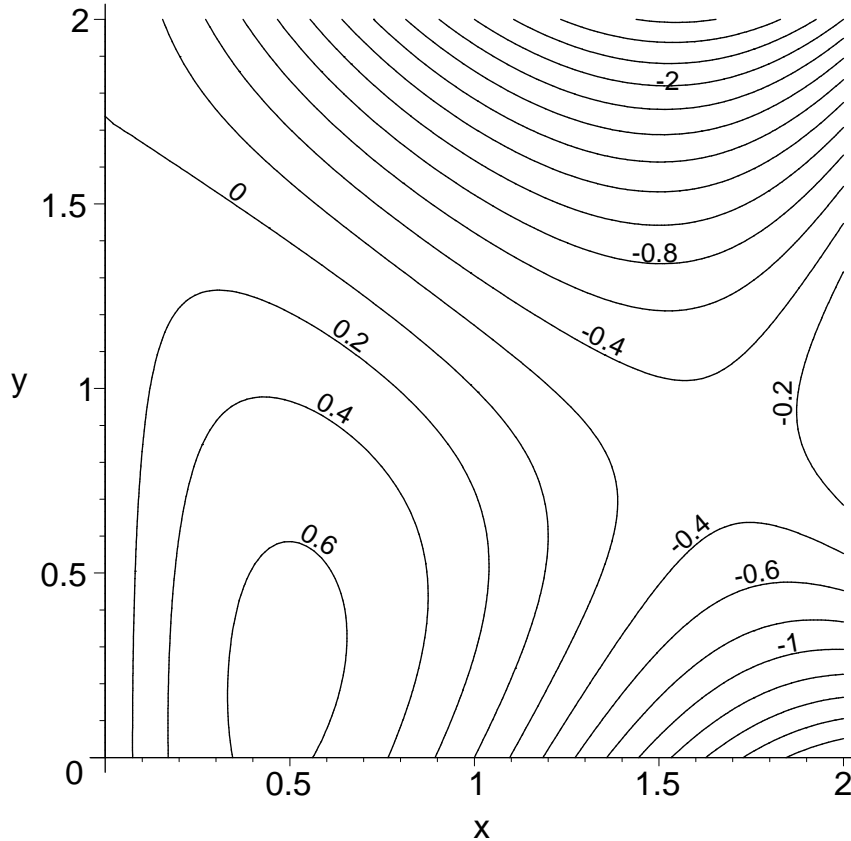
**Problem 2.** (Tuesday, 4 points: 3+1)

a) Let  $w = x^y (= e^{y \ln x})$ . Give an approximate formula for the small change  $\Delta w$  in the value of  $w$  produced by small changes  $\Delta x$  and  $\Delta y$  in the values of  $x$  and  $y$ . Use this to calculate an approximate value of  $1.98^{2.01}$ . Compare your answer with the exact value.

b) Starting from  $2^2 = 4$ , is the value of  $x^y$  more sensitive to small changes in  $x$  or in the exponent  $y$ ? (Justify your answer.)

**Problem 3.** (Tuesday, 7 points: 2+2+0+3)

The figure below is the contour plot of a function of two variables  $f(x, y)$ , for  $x$  and  $y$  ranging between 0 to 2 (scale: 1 unit = 5 cm; spacing between contour levels: 0.2).



a) Use the contour plot to determine whether  $f_x$  and  $f_y$  are  $> 0$ ,  $= 0$ , or  $< 0$  at the point  $(1, 1.5)$ . Same question at the point  $(1.2, 0.6)$ ?

b) The function plotted on the figure is  $f(x, y) = x^3 - xy^2 - 4x^2 + 3x + x^2y$ . Calculate the actual values of the partial derivatives at  $(1, 1.5)$  and  $(1.2, 0.6)$ .

c) Use the applet on the course web page (<http://math.mit.edu/18.02/applets/>) to plot the graph of  $f$  corresponding to this contour plot. Read briefly the directions on the web page, and familiarize yourself with the controls. In the “Level curves” view, see how the slider in the lower-left corner moves the highlighted contour line on both plots, and experiment with it in order to perfect your understanding of the contour plot. In the “Partial derivatives” view, see how clicking on a point in the small contour plot displays the  $x$ - and  $y$ -slices of the graph through that point. Finally, use the applet to check the answers you found in (b).

d) Use the applet to find two points where  $f_x = f_y = 0$ , and give their approximate coordinates. What happens to the level curves of  $f$  through these points? For each of the two points, describe what happens when you move towards North, South, East, West: does the value of  $f$  go up, down, or does it stay exactly the same?